MATHEMATICAL AND COMPUTATIONAL SCIENCES
DIVISION COLLOQUIUM

NONLINEAR MODELS WITH ANOMALOUS DIFFUSION

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ABSTRACT

There is ample physical motivation justifying consideration of nonlinear evolution equations with anomalous diffusion, i.e., different from the classical Brownian case. The simplest operators describing anomalous diffusion are the fractal operators of the form $-a_0\Delta + \sum a_j(-\Delta)^{\alpha_j/2}$, $0 < \alpha_j < 2$, where the fractional powers of the Laplacian correspond to the Lévy α-stable stochastic processes. We consider various generalizations of the Burgers equation $u_t - u_{xx} + (u^2)_x = 0$ used in hydrodynamics and statistical mechanics, e.g.

$$u_t - u_{xx} + (-\partial^2/\partial x^2)^{\alpha/2}u + (u^2)_x = 0,$$

$$u_t + (-\Delta)^{\alpha/2}u + f(u)_x = 0,$$

$$u_t + (-\Delta)^{\alpha/2}u + \nabla\cdot(uB(u)) = 0,$$

$$u_t + Lu + \nabla\cdot F(u) = 0,$$

where f and F are nonlinear functions, B is a linear integral operator and L is a general Lévy operator.

Besides standard mathematical questions such as solvability of the initial value problem for these equations and uniqueness and regularity of solutions, we study existence of special solutions (traveling waves or self-similar) and large time behavior of general solutions. Completely new phenomena (compared to the Burgers equation) appear, e.g. time asymptotics are linear in the first approximation, and only the second term of the asymptotic expansion reflects the nonlinear effects.

Connections with probability theory, stochastic differential equations driven by Lévy processes, and approximation by systems of interacting particles are also studied.

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